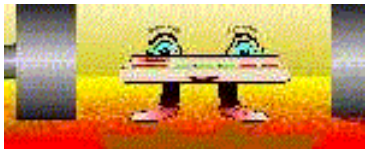
Lempel-Ziv-Welch algorithm



- Adaptive compression – no a priory knowledge of the symbol probabilities
- If the probability of symbols is known – use Huffman !
- Optimality of Huffman codes have a few assumptions : prefix-free codes, a known probability distribution and the symbols in the message are I.I.D. random variables.
- Hufmman codes do not necessarily reach the entropy bound.
- Solution – increase the number of symbols (alphabet) but the complexity of implementation is increased.

LZW- algorithm motivation

- Symbol probabilities change from message to message or within a single message
- **Standard approach** – take a few books in English and estimate the probabilities of the letters (symbols) – apply Huffman encoding
- Letter „x“ is not frequent but in the sentence … nothing is certain but death and ta_ _ _! The next letter is almost certainly „x“ !

- One approach to adapt to varying symbol probabilities is to use two pass method: measure the probability of symbols of given file and then use Huffman … (expensive and not clever w.r.t. group of symbols – what size …)
- **SOLUTION** – LZW adaptive encoding

LZW Compression

- Character sequences in the original text are replaced by codes that are dynamically determined.

- The code table is not encoded into the compressed text, because it may be reconstructed from the compressed text during decompression.

LZW Compression

- Assume the letters in the text are limited to {a, b}.
  - In practice, the alphabet may be the 256 character ASCII set.
- The characters in the alphabet are assigned code numbers beginning at 0.
- The initial code table is:

code	0	1
key	a	b

LZW Compression

code	0	1
key	a	b

- Original text = abababbabaabbabbaabba
- Compression is done by scanning the original text from left to right.
- Find longest prefix p for which there is a code in the code table.
- Represent p by its code pCode and assign the next available code number to pc, where c is the next character in the text that is to be compressed.

LZW Compression

code	0	1	2
key	a	b	ab

- Original text = abababbabaabbabbaabba
- p = a
- pCode = 0
- c = b
- Represent a by 0 and enter ab into the code table.
- Compressed text = 0

LZW Compression

code	0	1	2	3
key	a	b	ab	ba

- Original text = abababbabaabbabbaabba
- Compressed text = 0
- p = b
- pCode = 1
- c = a
- Represent b by 1 and enter ba into the code table.
- Compressed text = 01

LZW Compression

code	0	1	2	3	4
key	a	b	ab	ba	aba

- Original text = abababbabaabbabbaabba
- Compressed text = 01
- p = ab
- pCode = 2
- c = a
- Represent ab by 2 and enter aba into the code table.
- Compressed text = 012

LZW Compression

code	0	1	2	3	4	5
key	a	b	ab	ba	aba	abb

- Original text = ababababbabaabbbabbaabba
- Compressed text = 012
- p = ab
- pCode = 2
- c = b
- Represent ab by 2 and enter abb into the code table.
- Compressed text = 0122

LZW Compression

code	0	1	2	3	4	5	6
key	a	b	ab	ba	aba	abb	bab

- Original text = abababbabaabbabbaabba
- Compressed text = 0122
- p = ba
- pCode = 3
- c = b
- Represent ba by 3 and enter bab into the code table.
- Compressed text = 01223

LZW Compression

code	0	1	2	3	4	5	6	7
key	a	b	ab	ba	aba	abb	bab	baa

- Original text = abababbabaabbbabbaabba
- Compressed text = 01223
- p = ba
- pCode = 3
- c = a
- Represent ba by 3 and enter baa into the code table.
- Compressed text = 012233

LZW Compression

code	0	1	2	3	4	5	6	7	8
key	a	b	ab	ba	aba	abb	bab	baa	abba

- Original text = abababbabaabbabbaabba
- Compressed text = 012233
- p = abb
- pCode = 5
- c = a
- Represent abb by 5 and enter abba into the code table.
- Compressed text = 0122335

LZW Compression

code	0	1	2	3	4	5	6	7	8	9
key	a	b	ab	ba	aba	abb	bab	baa	abba	abbaa

- Original text = abababbabaabbabbaabba
- Compressed text = 0122335
- p = abba
- pCode = 8
- c = a
- Represent abba by 8 and enter abbaa into the code table.
- Compressed text = 01223358

LZW Compression

code	0	1	2	3	4	5	6	7	8	9
key	a	b	ab	ba	aba	abb	bab	baa	abba	abbaa

- Original text = abababbabaabbabbaabba
- Compressed text = 01223358
- p = abba
- pCode = 8
- c = null
- Represent abba by 8.
- Compressed text = 012233588

Code Table Representation

code	0	1	2	3	4	5	6	7	8	9
key	a	b	ab	ba	aba	abb	bab	baa	abba	abbaa

- Dictionary.
  - Pairs are (key, element) = (key,code).
  - Operations are : get(key) and put(key, code)
- Limit number of codes to 2^{12}.
- Use a hash table.
  - Convert variable length keys into fixed length keys.
  - Each key has the form pc, where the string p is a key that is already in the table.
  - Replace pc with (pCode)c.

Code Table Representation

code	0	1	2	3	4	5	6	7	8	9
key	a	b	ab	ba	aba	abb	bab	baa	abba	abbaa

code	0	1	2	3	4	5	6	7	8	9
key	a	b	0b	1a	2a	2b	3b	3a	5a	8a

LZW Decompression

code	0	1
key	a	b

- Original text = abababbabaabbabbaabba
- Compressed text = 012233588
- Convert codes to text from left to right.
- 0 represents a.
- Decompressed text = a
- pCode = 0 and p = a.
- p = a followed by next text character (c) is entered into the code table.

LZW Decompression

code	0	1	2
key	a	b	ab

- Original text = abababbabaabbabbaabba
- Compressed text = 012233588
- 1 represents b.
- Decompressed text = ab
- pCode = 1 and p = b.
- lastP = a followed by first character of p is entered into the code table.

LZW Decompression

code	0	1	2	3
key	a	b	ab	ba

- Original text = abababbabaabbabbaabba
- Compressed text = 012233588
- 2 represents ab.
- Decompressed text = abab
- pCode = 2 and p = ab.
- lastP = b followed by first character of p is entered into the code table.

LZW Decompression

code	0	1	2	3	4
key	a	b	ab	ba	aba

- Original text = ababababbabaabbbabbaabba
- Compressed text = 012233588
- 2 represents ab
- Decompressed text = ababab.
- pCode = 2 and p = ab.
- lastP = ab followed by first character of p is entered into the code table.

LZW Decompression

code	0	1	2	3	4	5
key	a	b	ab	ba	aba	abb

- Original text = abababbabaabbabbaabba
- Compressed text = 012233588
- 3 represents ba
- Decompressed text = abababba.
- pCode = 3 and p = ba.
- lastP = ab followed by first character of p is entered into the code table.

LZW Decompression

code	0	1	2	3	4	5	6
key	a	b	ab	ba	aba	abb	bab

- Original text = abababbabaabbabbaabba
- Compressed text = 012233588
- 3 represents ba
- Decompressed text = abababbaba.
- pCode = 3 and p = ba.
- lastP = ba followed by first character of p is entered into the code table.

LZW Decompression

code	0	1	2	3	4	5	6	7
key	a	b	ab	ba	aba	abb	bab	baa

- Original text = abababbababaabbabbaabba
- Compressed text = 012233588
- 5 represents abb
- Decompressed text = abababbabaabb.
- pCode = 5 and p = abb.
- lastP = ba followed by first character of p is entered into the code table.

LZW Decompression

code	0	1	2	3	4	5	6	7	8
key	a	b	ab	ba	aba	abb	bab	baa	abba

- Original text = abababbabaaabbabbaabba
- Compressed text = 012233588
- 8 represents ???
- When a code is not in the table, its key is lastP followed by first character of lastP.

- lastP = abb
- So 8 represents abba.

LZW Decompression

code	0	1	2	3	4	5	6	7	8	9
key	a	b	ab	ba	aba	abb	bab	baa	abba	abbaa

- Original text = ababababbaabbabbaabba
- Compressed text = 012233588
- 8 represents abba
- Decompressed text = ababababbaabbabbaabba.
- pCode = 8 and p = abba.
- lastP = abba followed by first character of p is entered into the code table.

Code Table Representation

code	0	1	2	3	4	5	6	7	8	9
key	a	b	ab	ba	aba	abb	bab	baa	abba	abbaa

- Dictionary.
  - Pairs are (key, element) = (code, what the code represents) = (code, codeKey).
  - Operations are : get(key) and put(key, code)
- Keys are integers 0, 1, 2, …
- Use a 1D array codeTable.
  - codeTable[code] = codeKey.
  - Each code key has the form pc, where the string p is a code key that is already in the table.
  - Replace pc with (pCode)c.

Time Complexity

- Compression.
  - O(n) expected time, where n is the length of the text that is being compressed.
- Decompression.
  - O(n) time, where n is the length of the decompressed text.