MATHEMATICAL METHODS

TUTORIAL 1

Problem 1. Theoretical questions:

(1) State and prove De Morgan's laws for two propositions.
(2) Prove that $(p \Rightarrow q) \wedge p \Rightarrow q$ is a tautology.
(3) Prove that $\neg\,(\forall x \in D_1.\ \exists y \in D_2.\ p(x,y)) \iff \exists x \in D_1.\ \forall y \in D_2.\ \neg p(x,y)$.

Problem 2. Theoretical questions:

(1) How we describe or define a set?
(2) What it means that A is a subset of B?
(3) What is the cardinality of a finite set?
(4) What is the power set of a given set?

Problem 3. Determine the cardinality of the following sets:

(1) $A = \{1, 2, 3, 1, 3\}$.
(2) $B = \{\{1, 2, 3, 1, 3\}, 2\}$.
(3) $C = \{\} = \varnothing$.
(4) $D = \{\{\varnothing\}\}$.

Problem 4. Determine, if A and B are equal sets:

(1) A is the set of all equilateral triangles and B is the set of all triangles with equal internal angles.
(2) A is the set of all triangles with two right angles and B is the set of all solutions of $x^2 + 1 = x^2$.
(3) A is the set of all rectangles and B is the set of all parallelograms with equal diagonals.

Problem 5. Define the *symmetric difference* of two sets A and B as $A \triangle B = (A \setminus B) \cup (B \setminus A)$. Prove the following:

(1) $A \triangle A = \varnothing$.
(2) $A \triangle \varnothing = A$.
(3) $A \triangle B = (A \cup B) \setminus (A \cap B)$.
(4) $A \triangle B = B \triangle A$.

Problem 6. By using the mathematical induction, prove the following:

(1)
$$\sum_{j=1}^{n} j(j+1) = \frac{1}{3} n(n+1)(2n+1).$$

(2)
$$\sum_{j=1}^{n} \frac{1}{\sqrt{j}} \geq \sqrt{n}.$$

(3) For every $n \in \mathbb{N}$, 3 divides $2^{2n} - 1$.

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