


$i \cdot i$

$i^3 = -i$

$i^4 = 1$

$i^5 = i$

...

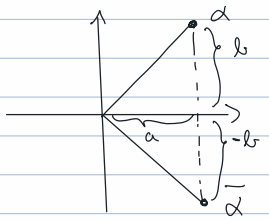
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$$(a,b) = (a,0)+(0,b) = \underbrace{(a,0)}_{\text{IZENAČIMO Z } a\in\mathbb{R}} + \underbrace{(b,0)}_{\text{IZENAČIMO Z } b\in\mathbb{R}} \cdot \underbrace{(0,1)}_{\text{:}} = a + b\cdot i$$

$\alpha = (a,b) = a + b\cdot i$ ← s števili zapisanimi v tej obliki računamo kot običajno, pri čemer $i^2$ vselej zamenjamo z $-1$.

$$(a+bi)\cdot(c+di) \overset{\text{DISTRIBUTIVNOST}}{=} ac + adi + bi\cdot c + \underbrace{bi\cdot di}_{-b\cdot d} = \underline{(ac-bd)+(a\cdot d+b\cdot c)\cdot i}$$

KONJUGIRANO ŠTEVILO: $\overline{\alpha} = \overline{a+bi} \overset{def}{=} a - bi$




$\overline{\alpha} = \alpha \iff \alpha \in \mathbb{R}$

$\overline{\alpha}\cdot\alpha = a^2 + b^2 \Rightarrow |\alpha| \overset{def}{=} \sqrt{\alpha\cdot\overline{\alpha}} = \sqrt{a^2+b^2}$

ABSOLUTNA VREDNOST $\alpha\in\mathbb{C}$

$$\alpha \cdot \frac{\overline{\alpha}}{a^2+b^2} = 1$$

$\alpha^{-1}$ INVERZ $\alpha\in\mathbb{C}$

$$\alpha^{-1} = \frac{a-b\cdot i}{a^2+b^2} = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2}\cdot i \quad ; \quad (a,b)^{-1} = \left(\frac{a}{a^2+b^2}, -\frac{b}{a^2+b^2}\right)$$

TRDITEV: $\alpha, \beta \in \mathbb{C}$ potem velja:

(i) $\overline{\alpha+\beta} = \overline{\alpha} + \overline{\beta}$

(ii) $\overline{\alpha\cdot\beta} = \overline{\alpha}\cdot\overline{\beta}$

(iii) $\alpha = a+b\cdot i$, $a = \underset{\text{REALNI DEL}}{\operatorname{Re}}\,\alpha$, $b = \underset{\text{IMAGINARNI DEL}}{\operatorname{Im}}\,\alpha \Rightarrow \boxed{a = \frac{\alpha+\overline{\alpha}}{2},\ b = \frac{\alpha-\overline{\alpha}}{2i}}$

TRDITEV: (LASTNOSTI ABSOLUTNE VREDNOSTI KOMPLEKSNIH ŠTEVIL)

$\alpha, \beta \in \mathbb{C}$, velja: (i) $|\alpha| \geq 0$, $|\alpha| = 0 \Leftrightarrow \alpha = 0$

(ii) $|\overline{\alpha}| = |\alpha|$

(iii) $|\alpha \cdot \beta| = |\alpha| \cdot |\beta|$

(iv) $|\mathrm{Re}\,\alpha| \leq |\alpha| \wedge |\mathrm{Im}\,\alpha| \leq |\alpha|$

(v) $|\alpha + \beta| \leq |\alpha| + |\beta|$ → TRIKOTNIŠKA NEENAKOST

DOKAZ: (ii) $|\overline{\alpha}| = \sqrt{\overline{\alpha} \cdot \overline{\overline{\alpha}}} = \sqrt{\alpha \cdot \overline{\alpha}} = |\alpha|$ ($\overline{\overline{\alpha}} = \alpha$)

(iv) $\alpha = a + bi$: $|\alpha|^2 = a^2 + \underbrace{b^2}_{\geq 0} \geq a^2 \overset{\sqrt{\ }}{\Rightarrow} |\alpha| \geq |a| = |\mathrm{Re}\,\alpha|$

IMAGINARNO ŠTEVILO: $\alpha = b \cdot i$, $b \in \mathbb{R}$

(v) $|\alpha + \beta|^2 = (\alpha + \beta) \cdot \overline{(\alpha + \beta)} = (\alpha + \beta) \cdot (\overline{\alpha} + \overline{\beta})$

$= \underbrace{\alpha \cdot \overline{\alpha}}_{|\alpha|^2} + (\alpha \cdot \overline{\beta} + \beta \cdot \overline{\alpha}) + \underbrace{\beta \overline{\beta}}_{|\beta|^2} = |\alpha|^2 + 2 \cdot \mathrm{Re}(\alpha \cdot \overline{\beta}) + |\beta|^2$

$2 \cdot ? = (\alpha \cdot \overline{\beta}) + \overline{(\alpha \cdot \overline{\beta})}$, $\frac{\square + \overline{\square}}{2} = \mathrm{Re}\,\square$

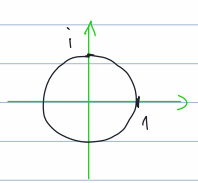
$\leq |\alpha|^2 + 2 \cdot |\alpha \cdot \overline{\beta}| + |\beta|^2 = |\alpha|^2 + 2 \cdot |\alpha| \cdot |\overline{\beta}| + |\beta|^2$ ($|\overline{\beta}| = |\beta|$)

$= (|\alpha| + |\beta|)^2$

$\Rightarrow |\alpha + \beta|^2 \leq (|\alpha| + |\beta|)^2$

$|\alpha + \beta| \leq |\alpha| + |\beta|$ ∎

OPOMBA: 1° $S = \{\alpha \in \mathbb{C} \mid |\alpha| = 1\}$ ⇒ JE GRUPA ZA $\cdot$

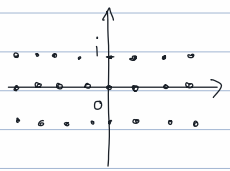



$|\alpha \cdot \beta| = |\alpha| \cdot |\beta| = 1 \cdot 1 = 1$

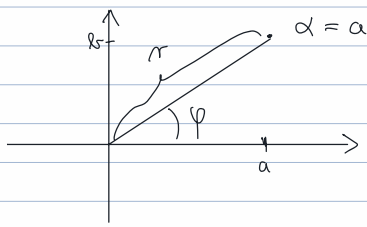
$\alpha, \beta \in \mathbb{C}, S$

$\alpha \in S$: $\alpha \cdot \overline{\alpha} = 1 \Rightarrow \overline{\alpha} = \alpha^{-1}$

2° KOLOBAR GAUSSOVIH ŠTEVIL: $G = \{ a + b \cdot i \in \mathbb{C} \mid a, b \in \mathbb{Z} \}$




## POLARNI ZAPIS KOMPLEKSNEGA ŠTEVILA




$\alpha = a + b \cdot i$

$r = |\alpha| = \sqrt{a^2 + b^2}$

$a = r \cdot \cos\varphi \quad \left( \cos\varphi = \frac{a}{r} \right)$

$b = r \cdot \sin\varphi$

$$\alpha = a + b \cdot i = r \cdot \cos\varphi + r \cdot \sin\varphi \cdot i = r \left( \cos\varphi + i \sin\varphi \right)$$

POLARNI ZAPIS $\alpha$: $\alpha = |\alpha| \cdot (\cos\varphi + i \cdot \sin\varphi)$

$r = |\alpha|$ = MODUL ŠT $\alpha$

$\varphi = Arg(\alpha)$ = ARGUMENT ŠT. $\alpha$

$0 \leq \varphi < 2\pi$

$\beta \in \mathbb{C}$: $\beta = |\beta| \cdot (\cos\psi + i \cdot \sin\psi)$ (PSI)

$$\alpha \cdot \beta = |\alpha| \cdot |\beta| \cdot (\cos\varphi + i \cdot \sin\varphi) \cdot (\cos\psi + i \cdot \sin\psi)$$

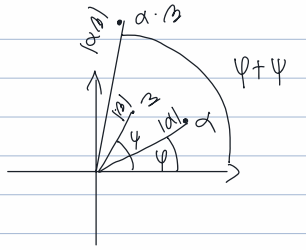
$$(\cos\varphi \cdot \cos\psi - \sin\varphi \cdot \sin\psi) + i (\cos\varphi \cdot \sin\psi + \cos\psi \cdot \sin\varphi)$$

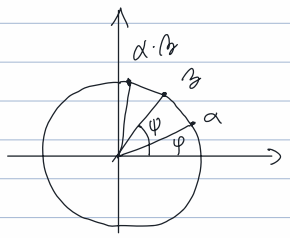
$\cos(\varphi + \psi)$ $\qquad$ $\sin(\varphi + \psi)$

$$\alpha \cdot \beta = |\alpha \cdot \beta| \cdot (\cos(\varphi + \psi) + i \cdot \sin(\varphi + \psi))$$

↓ MNOŽENJE JE ROTACIJA $\alpha$ ZA $\beta$

PRODUKT KOMPLEKSNIH ŠT. DOBIMO TAKO, DA ABSOLUTNE VREDNOSTI ZMNOŽIMO, ARGUMENTE PA SEŠTEJEMO, ČE DOBIMO KOT MED $2\pi$ IN $4\pi$ OD NJEGA ODŠTEJEMO $2\pi$



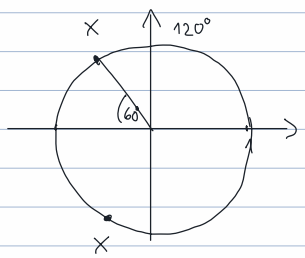

$\alpha, \beta \in S$

IZREK: MOIVROVA FORMULA: $(\cos\varphi + i\cdot\sin\varphi)^m = \cos(m\cdot\varphi) + i\cdot\sin(m\varphi)$

ZGLED: POIŠČITE REŠITVE ENAČBE $x^3 = 1 \Rightarrow x = 1$

$\| $

$x\cdot x\cdot x$




$x = -\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot i$

OPOMBA: $\varphi \longrightarrow \underbrace{\cos\varphi + i\cdot\sin\varphi}_{\in S} \ (= e^{i\varphi})$ $\qquad e^{i\varphi}\cdot e^{i\psi} = e^{i(\varphi+\psi)}$

VSOTA ŠT. $\varphi$ IN $\psi$ V $\mathbb{R}$ $\Rightarrow$ PRODUKT PRIREJENIH KOMPLEKSNIH ŠTEVIL

$(\mathbb{R}, +) \longrightarrow (S, \cdot)$ TAKA PRESLIKAVA SE IMENUJE HOMOMORFIZEM GRUP