MATHEMATICAL METHODS

TUTORIAL 3

Problem 1. Write the first five terms of the sequence $a_n = 3^{-n}\left(3^{n+1} - 8\right)$, and prove that it is strictly increasing and bounded. Calculate its infimum and supremum, and also its limit.

Problem 2. Calculate the following limits:

(1) $\lim_{n\to\infty}\left(\sqrt{n^2 - 4n} - \sqrt{n^2 + 3n}\right)$;
(2) $\lim_{n\to\infty}\left(4^{n-2} - 2\cdot 3^{n+2} + 2n^4\right) / \left(4^{n-1} + 2^{2n-3}\right)$.

Problem 3 (Harder)**.** Prove that the *harmonic series*

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots$$

diverges.

Problem 4. At the beginning of each month we made a deposit of the same amount of money. At the end of the eighth month since making deposits, we had 30000 EUR. Our bank uses 0.8333% monthly interest rate. Calculate the amount of our monthly deposit.

Date: 11. 11. 2026.