Programming

Recursive functions

5th lecture

Algorithm

- What is an algorithm?

1. It is a function that converts input to output

2. The algorithm calculates a result for each data, it always stops

Example

```
public int sestejDo0(int par) {
  int vsota;
  vsota = 0;
  while (par != 0) {
    vsota += par;
    par--;
  }
  return vsota;
} /* sestejDo0 */
```

- Is this an algorithm?

Sum numbers to 0

```
Int sestejDo0(int par) {
  int vsota;
  vsota = 0;                /* initialization to 0 */
  while (par != 0) {        /* while par is bigger than 0
  */
    vsota += par;           /* add par to sum */
    par--;                  /* decrease par */
  }
  return vsota;
} /* sestejDo0 */
```

And now for something completely different

```
Int sestejDo0(int par) {
  int vsota;
  vsota = 0;
  if (par <= 0)
    return 0;
  else {
    while (par != 0) {
      vsota += par;
      par--;
    }
    return vsota;
  }
} /* sestejDo0 */
```

```
Int sestejDo0(int par) {
  if (par <= 0)
    return 0;
  else
    return
       sestejDo0(par-1)+par;
} /* sestejDo0 */
```

What the function really does

$$\text{rezultat} = \left(\sum_{i=0..par} i\right) = \left(\sum_{i=0..par-1} i\right) + par$$

```
Int sestejDo0(int par) {
 if par is smaller or
   equalt to 0, the
   result is 0;
 else
   the result is equal to
    the sum of numbers
   from 0 to par-1,
   and add par
} /* sestejDo0 */
```

```
Int sestejDo0(int par) {
   if (par <= 0)

      return 0;
   else
      return
         sestejDo0(par-1)
         + par;
} /* sestejDo0 */
```

Recursive definition

```
Int sestejDo0(int par) {
  if (par <= 0)
    return 0;
  else
    return
       sestejDo0(par-1)+par;

} /* sestejDo0 */
```

```
Int sestejDo0(int par) {
  if (par <= 0)
    result is 0;
  else
    result is sum of the
   smaller problem and par
} /* sestejDo0 */
```

- Stoping condition
- Step divide and conquer (divide et impera) (**deli in vladaj)**

Again - algorithm

```
Int sestejDo0(int par) {
  if (par <= 0)
    return 0;
  else
    return
       sestejDo0(par-1)+par;
} /* sestejDo0 */
```

- Is this recursive function an algorithm?
- Yes, it stops for every value of `par`.
- *Proof* (induction on parameter `par`):
  - *Basis:* for each `par` <= 0 from the source code
  - *Hipothesys:* if the statement `par` = n-1 is true, than it is true also for `par` = n
  - *Step:* say, the function stops for `par-1`; then it ceartainly stops for `par` (from the source code)

Recursion and iteration

- Each iteration can be translated into recursion

```
public static void print(int[] polje) {
  for(int i = 0; i < polje.length; i++)
  {
    System.out.print(polje[i]);
    System.out.print(" ");
  }
  System.out.println();
} // print
```

Output all elements of the array

```
public static void print(int[] polje) {
  for(int i = 0; i < polje.length; i++)
    System.out.print(polje[i] + " ");
  System.out.println();
} // print
```

```
public static void print(int[] polje, int i) {
  if (i >= polje.length)
    System.out.println();
  else {
    System.out.print(polje[i] + " ");
    print(polje, i+1);
  }
} // print
```

Output all elements of the array

- if we come to the end of the array, print last element, else
- print the next character (the first in the current array) and the rest of the field

```
public static void print(int[] polje, int i)
{
  if (i >= polje.length)
    System.out.println();
  else {
    System.out.print(polje[i] + " ");
    print(polje, i+1);
  }
} // print
```

Search of an element

```
public static boolean
member(int[] polje, int elt) {
  for(int i = 0; i < polje.length; i++)
    if (polje[i] == elt) return true;
  return false;
} // member
```

```
public static boolean
member(int[] polje, int elt, int i) {
  if (polje[i] == elt)    return true;
  if (i >= polje.length)  return false;
  return member(polje, elt, i+1);
} // member
```

Search of an element

- if we find the element, return true
- if we come to the end of the array, return false
- else search the rest of the array

```
public static boolean
member(int[] polje, int elt, int i) {
  if (polje[i] == elt)    return true;
  if (i >= polje.length)  return false;
  return member(polje, elt, i+1);
} // member
```

Search for the biggest element

- If we are at the end of the array, return the biggest element, else:
- look at whether the current element is bigger than previously found element and, if so, this element becomes the new current biggest element
- then look in the rest of the field, if there a bigger element

Search for the biggest element

```
public static
int maximum(int[] polje, int i, int trenMaks) {
  if (i >= polje.length)  return trenMaks;
  if (polje[i] > trenMaks) trenMaks = polje[i];
  return maximum(polje, i+1, trenMaks);
} // maximum
```

- Call like this:

what about `maximum(polje, 0, polje[0])`

this call? `maximum(polje, 1, polje[0])`?

Search for the biggest element - 2.

- If this is the last item in the array, then this is the biggest element in the array, otherwise:
- find the biggest element in the remainder of the array
- look at whether the current element is bigger than the current biggest element in the remainder of the field and, if so, we claim this is the current biggest element

Search for the biggest element - 2.

```
public static int maximum(int[] polje, int i) {
  int maksPreost;
  if ((i+1) == polje.length)  return polje[i];
  maksPreost = maximum(polje, i+1);
  if (polje[i] > maksPreost) maksPreost = polje[i];
  return maksPreost;
} // maximum
```

- Call like this:

```
maximum(polje, 0)
```