1 Pumping lemma for context-free languages

The pumping lemma for context-free languages (Bar-Hillel lemma): Let L be a CFL, then there exists $p > 0$ such that if $z \in L$ and $|z| \geq p$, then z can be written as $uvwxy$ where

1. $|vwx| \leq p$
2. $vx \neq \varepsilon$
3. $uv^k wx^k y \in L \quad \forall k \geq 0$

Pumping Lemma for CFL can be used to prove that some languages are not context free.

Exercise 1:
$\Sigma = \{a, b, c\}$
$L = \{a^n b^n c^n \mid n \geq 0\}$

Exercise 2:
$\Sigma = \{a, b, c\}$
$L = \{a^n b^m c^n \mid n \geq m\}$

Exercise 3:
$\Sigma = \{a, b, c\}$
$L = \{a^i b^j c^k \mid i \leq j \leq k\}$

Exercise 4:
$\Sigma = \{a, b\}$
$L = \{ww \mid w \in \{a, b\}^*\}$

Exercise 5:
$\Sigma = \{a, b\}$
$L = \{a^p \mid p \text{ prime }\}$

2 Testing membership in a CFL

We can decide the if a string w is the member of a context free language L. An efficient way to do this is the dynamic-programming (table filling) CYK (Cocke-Younger-Kasami) Algorithm.

How can we decide if string $w = a_1a_2a_3a_4a_5$ is in L? We need the CNF grammar for L to do this, and fill out the following table.

x_{15}				
x_{14}	x_{25}			
x_{13}	x_{24}	x_{35}		
x_{12}	x_{23}	x_{34}	x_{45}	
x_{11}	x_{22}	x_{33}	x_{44}	x_{55}
a_1	a_2	a_3	a_4	a_5

- Each row of the table corresponds to a particular length of substrings of w.
- x_{ij} corresponds to the subset of variables that can generate substring $a_ia_{i+1}...a_j$
- We start by computing substrings of length 1.
- Substrings of length k will be generated by combining substrings of length $k - l$ and l (for all l values).
- The string w is a member of L, if $S \in x_{15}$

Exercise 1: Use the CYK algorithm to determine if $baaba \in L(G)$ holds, where the productions of your grammar G are

$S \rightarrow AB \mid BC$
$A \rightarrow BA \mid a$
$B \rightarrow CC \mid b$
$C \rightarrow AB \mid a$

Exercise 2: Use the CYK algorithm to determine if $cabab \in L(G)$ holds, where the productions of your grammar G are

$S \rightarrow AB \mid b$
$A \rightarrow CB \mid AA \mid a$
$B \rightarrow AS \mid b$
$C \rightarrow BS \mid c$

Exercise 3: Use the CYK algorithm to determine if $abaab \in L(G)$ holds, where the productions of your grammar G are

$S \rightarrow AB \mid SS \mid a$
$A \rightarrow BS \mid CD \mid b$
$B \rightarrow DD \mid b$
$C \rightarrow DE \mid a \mid b$
$D \rightarrow a$
$E \rightarrow SS$

Exercise 3: Use the CYK algorithm to determine if *she eats a fork with a fish* $\in L(G)$ holds, where $G = \{\{S, VP, PP, NP, V, P, N, D\}, \{a, eats, fish, fork, with\}, R, S\}$, and the productions of R are

$$
\begin{aligned}
S &\rightarrow NP\ VP \\
VP &\rightarrow VP\ PP \\
VP &\rightarrow eats \\
PP &\rightarrow P\ NP \\
NP &\rightarrow D\ N \\
V &\rightarrow eats \\
P &\rightarrow with \\
N &\rightarrow fish \\
N &\rightarrow fork \\
D &\rightarrow a
\end{aligned}
$$