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# MVAJ

## Middterm Exam

### Exercises

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Write out your solutions in a clear and precise manner.  
All problems are weighted equally.

**Exercise 1.** Show that for all integers  $n \geq 1$ ,

$$1 \cdot 2 + 2 \cdot 3 + \dots + n \cdot (n + 1) = \frac{n(n + 1)(n + 2)}{3}.$$

**Exercise 2.** Verify the binomial identity

$$\binom{n}{h} \binom{n-h}{k} = \binom{n}{k} \binom{n-k}{h}.$$

**Exercise 3.** Analyze the existence of integer solutions of the following Diophantine equations. Find all the solutions when possible :

(a)  $28x + 16y = 97$

(b)  $28x + 16y = 100.$

**Exercise 4.** Show that 7 divides  $5^{6n} - 1$  for every integer  $n \geq 1$ .

**Exercise 5.** Use the Sieve or Erastosthenes to find all primes at most 60.

**Exercise 6.** Prove that  $\sqrt{2}$  is not a rational number.