Information Economics and Robust Institutions

Jernej Čopič*

2018

1 Roadmap

– "What will this be about?"

Mostly, my research and related literature ("dynamic" may evolve):

Strategic environments with incomplete information; institutions which interactions to achieve maximal possible welfare (economic efficiency) ; robustness to details of informational assumptions, including knowledge of participants (who are rational) regarding other participants' rationality.

No knowledge of the details of the probability distribution over unknown (private) parameters.

1. Fundamentals.

Four important topics:

(i) Complete information: strategic concepts (intro);

(ii) Complete info: measurement of efficiency (intro);

(iii) Incomplete info: strategic concepts (intro) -¿ dominant strategies and other.

*Jernej Čopič is at Aix-Marseille Université, GREQAM, and a visiting researcher at CERGE-Ei. Email: jernej.copic@univ-amu.fr

(iv) Incomplete info: measurement of welfare (intro+)

2. Economic institutions.

"games" conceived to achieve certain results;

(i) Game of complete information (intro)

(ii) Game of incomplete information (intro)

(iii) Revelation principle (dominant strategies) and direct revelation mechanisms (intro+)

(iv) A more general version of revelation principle (maybe)

3. Side questions (for now): knowledge and rationality

4. Application(s) (most): **Auction(s)**, Double auction, Public good provision...

2 Complete information: environments, games, solution concepts, welfare

For the purpose of this section all of the following concepts and considerations are common knowledge among all entities (defined below) including the entities themselves. Also, for now these environments are all cardinal, in that the individuals have utility functions rather than simply preference rankings (in which case such an environment would be ordinal).

Environment with complete information is given by (N, Y, u, c); c is optional, in particular, when $c \equiv 0$ (see below) we can omit it from the description of the environment in which case the environment is given by (N, Y, u).

- $N = \{1, ..., n\}$ individuals (abbr: sg: indiv; pl: indivs), $i \in N$, $j \neq i, -i = N \setminus \{i\}$; until further notice (probably almost always) we will assume that N is finite; in some cases $N = \{0, 1, ...n\}$ – see example auction below.

- Y is the set of social allocations (abbr, sg: aloc, pl: alocs) where $Y = X$ (no transfers) or $Y = X \times \mathbb{R}^n$, most often X is finite or at least compact; X is the set of alternatives (sg: alt; pl: alts) and $\mathbb{R}^n$ are transfers (or prices); Mostly environments with transfers, where $y \in Y$: $y = (x, p), x \in X, p \in \mathbb{R}$ are prices (transfers); a negative $p_i < 0$ is a payment by indiv i and a non-negative $p_i \geq 0$ signifies a receipt by i;

- u utility functions; $u_i : Y \to \mathbb{R}$, "increasing," concave...

  Special case: separable utility[1], $u_i(y) = \nu_i(x) + \bar{u}_i(p)$;

  Important special case – quasi-linear environment: $u_i(y) = \nu_i(x) + p_i$.

- c is the cost function in environments with transfers – this is optional and is used to describe feasibility when convenient; $c : X \to \mathbb{R}$; when $c(x) < 0$ then c can be interpreted as the social cost of alt x; when $c(x) \geq 0$ it is the aggregate social gain of social alternative x (this requires some entity outside the environment willing to pay numeraire into the system for the social alternative x, e.g., installing toxic waste facility).

  Key: Here, a $y \in Y$ is feasible, if it satisfies budget balance,

$$\sum_{i \in N} p_i \leq c(x) \tag{1}$$

  It satisfies exact budget balance when above is an equality.

- VERY IMPORTANT: RANDOMIZATION; CONVENTIONS (abuse of notation). Whenever appropriate (which is most of the time), Y also describes the set of randomized alocs,

[1](shitty, possibly inconsistent notation of $\bar{u}$ bc we will almost never work with this here anyways, meant more as an example, if it becomes an issue later revise)

or lotteries over alocs; E.g., in environment w/ transfers y is a lottery over $X \times \mathbb{R}^n$.

$\omega_y = (\omega_x, \omega_p)$ is then a realization from that lottery;[2]

Denote by *support*(y) the support of the lottery y (see some probability theory/measure theory book, e.g., Durret, Rudin...).

$u_i(y)$ is then interpreted as expected utility of i w.r.t. the lottery y,

$$u_i(y) = E_y[u_i(\omega_y)] = E_y[u_i(\omega_x, \omega_{p_i}). \tag{2}$$

Important: budget balance (as well as exact budget balance) can then take two forms.

Strong budget balance (exact when equality) – budget balance point-wise w.r.t. all realizations of lottery y:

$$\sum_{i \in N} \omega_{p_i} \leq c(\omega_x), \forall \omega_y \in \mathit{support}(y). \tag{3}$$

Weak budget balance (exact when equality) – budget balance required only on average w.r.t. lottery y:

$$\sum_{i \in N} E_y[\omega_{p_i}] \leq E_y[c(\omega_x)]. \tag{4}$$

IMPORTANT: i will mostly write out the definition for deterministic case and the definition for randomized case follows from the conventions adopted herein.

Several examples of environments that will be used throughout (here described for the case of complete information, mostly with transfers, to be appended later):

- Trivial environment (useful to think about the most trivial possible)

- Bargaining (bilateral trade) over one indivisible good (with complete information this is equivalent to bargaining over one unit of surplus, i.e., divisible): $N = \{1, 2\}$, usually 1 is

[2]important to note that $y = (x, p)$ should not be used then as this would suggest statistical independence, i.e., that y is a product of its marginal distributions over X and $\mathbb{R}^n$.

the seller, 2 is the buyer, with transfers, $X = \{1, 2\}$, $x = i$ means that indiv i gets the good; quasi-linear, $\nu_i(x) = \mathbf{1}_{\{x=i\}}$, where $\mathbf{1}_{\{x=i\}}$ is the indicator function,

$$\mathbf{1}_{\{x=i\}} = \begin{cases} 1, & \text{if } x = i, \\ 0, & \text{if } x \neq i. \end{cases}$$

Therefore, $u_i(y) = \mathbf{1}_{\{x=i\}} + p_i$. Note here $c \equiv 0$. One could expand this example to allow for free disposal of the good

- Provision of an indivisible public good (equivalent to multilateral bargaining over one unit of surplus, after normalization), e.g., constructing a bridge, cost normalized to 1: $N = \{1, ..., n\}$, $X = \{0, 1\}$, quasi-linear, $\nu_i(x) = v_i \mathbf{1}_{\{x=1\}}$, where $v_i \in [0, 1]$ is the value of the public good to the indiv i; $c(0) = 0$, $c(1) = 1$.

- Auction of one indivisible commodity (good) with zero production cost: $N = \{0, 1, ..., n\}$, $i = 0$ is the seller, $X = N$, $x = 0$ means the good is unsold, quasi-linear, $\nu_i(x) = v_i \mathbf{1}_{\{x=i\}}$, $v_0 = 0$, $v_i \in [0, 1]$, $i > 0$, $c \equiv 0$. If convenient and causes no confusion we can also exclude the seller from this description.

Measures of efficiency, or welfare evaluations: Pareto efficiency, social welfare function.

- Let $y, y' \in Y$ both be feasible. y' Pareto dominates y if,

$$u_i(y') \geq u_i(y), \forall i \in N, \text{ with at least one strict inequality.} \tag{5}$$

  Notation: $y' \succ y$.

- $y \in Y$, y feasible, is Pareto efficient (or classically efficient or efficient) if $\nexists y' \in Y$, such that y' is feasible and $y' \succ y$.

- Let $\lambda \in \mathbb{R}^N$, $\lambda_i \geq 0$, and define the utilitarian social welfare function $W^\lambda(.)$ by,

$$W^\lambda(y) = \sum_{i \in N} \lambda_i u_i(y), y \in Y. \tag{6}$$

- For a set B denote by $cl(B)$ its closure.[3] We have the following theorem (elementary to prove, well known, see Wilson,...).

Theorem 1. *Let Y be convex and u_i increasing and concave. Then,*

$$cl(\{y \in Y \mid y = \arg \max_{y' \in Y, y' \text{ feasible}} W^\lambda(y'), \lambda \in \mathbb{R}^N_+\} = \{y \in Y \mid y \text{ Pareto efficient}\}.$$

- A special case is the egalitarian social welfare function where $\lambda_i = 1, \forall i$.

  Another special case is maximizing the utility of a particular indiv j in which case $\lambda_j = 1$, $\lambda_i = 0, \forall i \neq j$.

- In a quasi-linear environment the alt x, which maximizes any social welfare function (under "reasonable" feasibility requirements) is always the same – the one that maximizes the so-called social surplus (can prove this as an exercise).

A game of complete information.[4]

In an environment (N, Y, u, c) a game is given by (A, γ); Or, a game in an environment is given by (N, Y, u, c, A, γ).

[3](appropriately defined in the context of the space in which B lives, should be unambiguous otherwise need to define the topology of that space etc.)

[4](here i will mostly only consider simultaneous-moves games and won't consider extensive-form games unless specified otherwise).

an indiv in a game is usually called *player*.

Note that this contrasts with most definitions in the literature (see below).

- $A = \times_{i \in N} A_i$, where A_i is the set of actions of indiv i, $a_i \in A_i$ is action of indiv i, $a \in A$ is a profile of actions.[5]

- $\gamma : A \to Y$ is the outcome mapping, so that $u(\gamma(a))$ are the utilities of the indivs when the action profile is $a \in A$.

  An interpretation of a game is that it is an institution which mediates interactions and thus leads to different allocations depending on the indivs' actions within the institution.

  Note that in the literature (see Myerson, Osborne and Rubinstein,...) a game is usually defined in reduced form by (N, A, u), where indivs' (or players') utilities are directly on profiles of actions.

  IMPORTANT: consistent with this we will use a shorthand abuse of notation: $u_i(a) = u_i(\gamma(a))$.

- IMPORTANT: Note that in general, γ may also map into randomized outcomes;

  it may also be that a is a randomization over actions in which case.

  Conventions regarding randomizations apply.

Strategies S and solution concepts.

- $S = \times_{i \in N} S_i$ is the set of indivs' strategies in a game, where $s_i \in S_i$ is the strategy of player i, $s \in S$ is a strategy profile.

  Here S_i is the set of lotteries over A_i.

  According to all the previous conventions we may use shorthand notation $u_i(s)$.

[5]Note that it is entirely without loss of generality to consider action set, which is a Cartesian product of indivs' action sets.

- A strategy profile s constitutes a dominant strategy equilibrium if,

$$u_i(s_i, s'_{-i}) \geq u_i(s'_i, s'_{-i}), \forall i, s'_i, s'_{-i}. \tag{7}$$

- A strategy profile s constitutes a Nash equilibrium if,

$$u_i(s) \geq u_i(s'_i, s_{-i}), \forall i, s'_i. \tag{8}$$

- Note that an outcome in a game needs to be appropriately defined (think); more about that later.

- We will also perhaps later define notions of correlated equilibrium in various ways under various circumstances.